**Letter**

**Adhesion-governed buckling of thin-film electronics on soft tissues**

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**HIGHLIGHTS**

- Adhesion-governed buckling physics for thin-film on elastomer.
- The transitions between buckling modes are predicted analytically.
- Mechanics discussed in the context of bio-integrated electronics applications.

**ABSTRACT**

Stretchable/flexible electronics has attracted great interest and attention due to its potentially broad applications in bio-compatible systems. One class of these ultra-thin electronic systems has found promising and important utilities in bio-integrated monitoring and therapeutic devices. These devices can conform to the surfaces of soft bio-tissues such as the epidermis, the epicardium, and the brain to provide portable healthcare functionalities. Upon contractions of the soft tissues, the electronics undergoes compression and buckles into various modes, depending on the stiffness of the tissue and the strength of the interfacial adhesion. These buckling modes result in different kinds of interfacial delamination and shapes of the deformed electronics, which are very important to the proper functioning of the bio-electronic devices. In this paper, detailed buckling mechanics of these thin-film electronics on elastomeric substrates is studied. The analytical results, validated by experiments, provide a very convenient tool for predicting peak strain in the electronics and the intactness of the interface under various conditions.

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that rely on intimate contact and coupling to the tissues. Detailed mechanics analysis of the buckling physics that accounts for any tissue stiffness and any interfacial adhesion is presented in this study to predict the intactness of the bio-electronics interfaces.

The various buckling modes in the previous work \[1,2,16–20\] can be categorized into the four modes shown in Fig. 1. Under none to minor compression, the film does not buckle and remains flat (Fig. 1(a)); as the compression increases, the film wrinkles into multiple small waves on top of the elastomer but does not delaminate from the interface, which we refer to as the wrinkling mode (Fig. 1(b)); under further compression, the multiple waves merge into one and cause the film to partially delaminate from the interface, which is the partial delamination mode (Fig. 1(c)); more compression eventually causes the film to delaminate totally from the interface, which we define as the total delamination mode in this study (Fig. 1(d)). The energies of these different buckling modes are formulated and then compared in the next section to explain transitions between them.

Here we consider a film-structure of length \(L\), thickness \(h\), and Young’s modulus \(E\) laminated on top of a soft substrate of Young’s modulus \(E_s\), and the work of adhesion for the interface is \(\gamma\), and the structure is under a compressive applied strain of \(|\varepsilon|\). By assuming a sinusoidal buckling shape of wavelength \(0 < l < L\) (Fig. 1(c)), Wang et al. \[23\] analyzed the energies for the flat, partial and total delamination modes. Their analysis is elaborated in the Supplementary Information and summarized in the following. All the energies are normalized by \(Eh^2\) for convenience, where \(\varepsilon_c = (\pi^2 h^2) / (3L^2)\). We also define the following non-dimensional quantities: the normalized applied strain \(\varepsilon = |\varepsilon| / \varepsilon_c\), the normalized critical wrinkling strain \(\varepsilon_w = (3E_s/E)^{2/3} / (4\varepsilon_c)\), the normalized adhesion \(g = \gamma / (8Eh^2)\) and the normalized delaminated length \(a = l/L\).

For the flat mode, the total energy of the system consists of the membrane energy of the film, and the adhesion energy of the entire interface, and is obtained as

\[U^\text{flat} = \frac{1}{2} \varepsilon^2 - 8g.\]  \(1\)

For the partial delamination mode, the total energy consists of the membrane and bending energy of the film and the adhesion energy of the un-delaminated part of the interface \([\text{length of } (L - l)]\), and is obtained as

\[U^\text{part delam} = ea^2 - \frac{1}{2} a^{-4} - 8g (1 - a).\]  \(2\)

Energy minimization with respect to \(a\) requires the first derivative of Eq. (2) to be zero and the second derivative to be greater than zero, therefore \(a\) can be solved from

\[\begin{align*}
4ga^5 - ea^2 + 1 &= 0, \\
\sqrt{3/ (5g)} &< a \leq 1,
\end{align*}\]  \(3\)

where \(a \leq 1\) is due to the constraint that \(l \leq L\).

For the total delamination mode, the energy consists of the membrane and bending energy of the film, and is obtained as

\[U^\text{tot delam} = e - \frac{1}{2}.\]  \(4\)

In this study, we find that a fourth buckling mode, i.e. the wrinkling mode, exists under certain conditions. Following similar approach of Jiang et al. \[2\], the energy of this mode consists of the membrane and bending energies of the film, the strain energy of the substrate, as well as the adhesion energy of the interface, and can be obtained analytically as

\[U^\text{wrinkle} = e^w \left( e - \frac{1}{2} e^w \right) - 8g.\]  \(5\)

It should be noted that this energy only exists when the applied strain exceeds the critical buckling strain, namely \(e > e_w\).

Here we adopt a typical case of \(e_w = 4\) and \(g = 3\) to facilitate the discussion. Figure 2 shows the four energy curves versus the normalized strain \(e\). All the curves are obtained analytically from Eqs. (1) to (5), except for the case of local buckling (blue curve). It is clearly shown in Fig. 2 that for very small strain \(e\), the flat mode has the lowest energy. As \(e\) increases, wrinkling, partial delamination and then total delamination modes become the lowest energy state in sequence. Intersections of the above energy curves are important because they indicate the transitions from one buckling mode to another. Depending on the values of \(e_w\) and \(g\), there are 6 possible intersections between these curves, which are found below.
The intersection between flat and wrinkling (black and green curves) is found by setting $U_{\text{flat}} = U_{\text{wrinkle}}$ (Eqs. (1) and (5)), which yields

$$e_{\text{flat-wrinkle}} = e_w.$$  

(6)

The intersection between flat and total delamination (black and red curves) is found by setting $U_{\text{flat}} = U_{\text{total delam}}$ (Eqs. (1) and (4)), which yields

$$e_{\text{flat-tot delam}} = 4\sqrt{g} + 1.$$  

(7)

The intersection between wrinkling and total delamination (black and red curves) is found by setting $U_{\text{wrinkle}} = U_{\text{total delam}}$ (Eqs. (4) and (5)), which yields

$$e_{\text{wrinkle-tot delam}} = \frac{16g - 1 + e_w^2}{2e_w - 2}.$$  

(8)

Following the analysis of Wang et al. [23], the intersection between flat and partial delamination is found to be

$$e_{\text{flat-partial delam}} = 5g^{2/5},$$  

(9)

and that between partial delamination and total delamination is

$$e_{\text{partial-tot delam}} = 4g + 1.$$  

(10)

The intersection between wrinkling and partial delamination cannot be obtained analytically because the energy of partial delamination needs to be solved numerically from Eqs. (2) and (3). Here an approximate solution is obtained. We notice that the blue curve for partial delamination is very close to a linear line, and two points on this line can be given analytically by Eqs. (9) and (10) and Eqs. (1) and (4) as

$$\begin{cases} 5g^{2/5} - \frac{25}{2}g^{4/5} - 8g \\ 4g + 1 \end{cases}$$

intersection of green and black curves,

$$\begin{cases} 4g + 1 \end{cases}$$

intersection of green and red curves.

The energy curve for partial delamination can be approximated by the straight line connecting the two points in Eq. (11). The intersection point between this line and the wrinkling curve can be then obtained analytically as

$$e_{\text{wrinkle-partial delam}} = \frac{f \cdot 5g^{2/5} - e_w^2 - 25g^{4/5}}{f - 2e_w},$$  

(12)

where $f = (24g + 1 - 25g^{4/5}) / (4g + 1 - 5g^{2/5})$. It is verified in the next sub-section that Eq. (12) agrees very well with numerical solution obtained by Eqs. (2), (3) and (5).

By carefully comparing the energies, one can determine which buckling mode has the lowest energy. However, the relations between these energies depend on the values of $e_w$ and $g$, and therefore require careful investigation of various cases. We categorize these cases by the value of $e_w = (3E_c / E)^{2/3} / (4e_c)$ (an indication of relative stiffness of the substrate) as the following.

1. $0 < e_w \leq 1$: for extremely soft substrate, it is found that the energy of the wrinkling mode is always lower than those of partial and total delamination modes. Therefore, the deformation map is obtained from Eq. (6) as

$$e_{\text{wrinkle}},$$  

when $e < e_w$.

$$e_{\text{flat}}$$  

when $e \geq e_w$.  

(13)

2. $1 < e_w \leq 3$: for soft substrate, it is found that the wrinkling mode only exists for relatively strong adhesion of $g > (e_w - 1)^2 / 16$ and partial delamination does not happen. For weak adhesion of $g \leq (e_w - 1)^2 / 16$, transition strain for flat- (total delamination) can be found by Eq. (7); for stronger adhesion, transition strain for flat–wrinkling and wrinkling–(total delamination) are given by Eqs. (6) and (8):

$$\begin{cases} g \leq (e_w - 1)^2 / 16, \quad \text{flat}, \quad e < 4\sqrt{g} + 1, \\ g > (e_w - 1)^2 / 16, \quad \text{total delamination}, \quad e \geq 4\sqrt{g} + 1, \\ g \leq e_w, \quad \text{wrinkle}, \quad e \leq 16g - 1 + e_w^2, \\ g > e_w, \quad \text{total delamination}, \quad e \geq 16g - 1 + e_w^2. \end{cases}$$  

(14)

3. $3 < e_w \leq 5$: the conclusions are the same as Case 2 for weak adhesion of $g \leq (e_w - 1)^2 / 8$ (Case 3). For stronger adhesion, the transitions from flat to wrinkling, then to partial and total delamination modes can be obtained from Eqs. (6), (10) and (12) as

$$\begin{cases} g \leq (e_w - 1)^2 / 16, \quad \text{same as Case 2}, \\ g > (e_w - 1)^2 / 16, \quad 8(e_w - 3) \leq g \leq (e_w - 1)^2, \\ e_w, \quad \text{flat}, \quad e < e_w, \\ g \leq e_w, \quad \text{wrinkle}, \quad e \leq e_{\text{wrinkle–partial delam}}, \\ g > e_w, \quad \text{partial delam}, \quad e \leq 8(e_w - 3), \\ g \leq e_w, \quad \text{total delamination}, \quad e \geq 8(e_w - 3), \\ g > e_w, \quad \text{total delamination}, \quad e \geq 4g + 1. \end{cases}$$  

(15)

(4) $e_w > 5$: for relatively stiffer substrate (note: its Young’s modulus is still four to five orders of magnitude lower than that of the film), the conclusions are the same as the results of Wang et al. [23] for weak adhesion of $g \leq (e_w / 5)^{2/7}$. For adhesion stronger than that, the transitions are the same as in Case 3:

$$\begin{cases} g \leq (e_w / 5)^{2/7}, \quad \text{same as Ref. [22]}, \\ g > (e_w / 5)^{2/7}, \quad \text{same as Case 3}. \end{cases}$$  

(16)

In summary, based on the relative stiffness of the elastomer [indicated by the value of $e_w = (3E_c / E)^{2/3} / (4e_c)$], the normalized adhesion $g = \gamma / (8Eh_c^2)$, and the normalized strain $e = |\varepsilon| / \varepsilon_c$, the buckling modes can be determined from Eqs. (13) to (16). These equations are plotted as the deformation maps $|\varepsilon|$ versus $g$, namely $|\varepsilon| / \varepsilon_c$ versus $\gamma / (8Eh_c^2)$ in Fig. 3. It should be noted that the range of $e_w$ determines the pattern of the map, and in
Fig. 3. Deformation maps ($|\varepsilon|/\varepsilon_c$ versus $\gamma/(8Eh^2)$) that separate the four buckling modes for various values of $e_w$; the plots are generated for representative $e_w$ values of (a) 0.8; (b) 2.5; (c) 4.0; (d) 7.0. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Each map the value of $e_w$ determines location of the black line. Figure 3 shows clearly the transitions of the four buckling modes under various conditions. It should be noted that the approximate analytical solution given by Eq. (12) (solid red line in Fig. 3(c), (d)) agrees very well with the results obtained numerically (red triangle dots) from Eqs. (2), (3) and (5).

Here we use the example in the previous studies [21,22] to show the utility of the presented results. The material and geometric and mechanical properties [22,23] are $E = 2.5$ GPa, $h = 1.4 \mu$m, $L = 150 \mu$m, $E_s = 2.0$ MPa, and $\gamma = 0.16$ J/m$^2$, which correspond to the normalized values of $e_w = 15.6$ and $g = 69.6$. Under these two conditions, Eq. (16) applies and gives the following results (these can also be obtained from Fig. 3(d)):

- **flat**, when $|\varepsilon| < 0.45%$,
- **wrinkling**, when $0.45% \leq |\varepsilon| < 0.92%$,
- **partial delam.**, when $0.92% \leq |\varepsilon| < 8.0%$,
- **total delam.**, when $|\varepsilon| \geq 8.0%$.

These results agree very well with experimental observation shown in Fig. 4: the film is **flat** (Fig. 4(a)) before compression is applied; under very small strain it **wrinkles** into multiple waves (Fig. 4(b)) and then quickly transits to the **partial delamination** mode (Fig. 4(c)); when $|\varepsilon|$ exceeds about 8.5% [23], the film **totally delaminated** from the substrate (Fig. 4(d)), which agrees very well with the 8.0% strain predicted by the analytical model. It should be noted that there may exist another buckling mode between the **wrinkling** and **partial delamination** modes, in which the film delaminates from the substrate from multiple locations. However, since the transitions happen at very similar strain levels, we propose to adopt the simplified model presented here.

The deformed shape of the film and the peak strain for the **flat**, **partial/totally delamination** modes are analyzed in detail by Wang et al. [23]. For the **wrinkling** mode, Jiang's analysis [2] shows that the wrinkling wavelength $\lambda$ and amplitude $A$ can be obtained by

$$
\begin{align*}
\lambda &= 2\pi h \left( \frac{E}{3E_s} \right)^{1/3}, \\
A &= h \sqrt{4|\varepsilon|} \left( \frac{E}{3E_s} \right)^{2/3} - 1, 
\end{align*}
$$

(17)
which gives the wavelength to be 67.5 µm and agrees reasonably with 56.8 µm from experiments (Fig. 4(b)). This predicts 2–3 waves over the total span of \( L = 150 \mu m \), which again agrees with experimental observations (only the middle wave of the three waves in Fig. 4(b) spans for an entire wavelength of \( \lambda \)). Therefore, for the wrinkling mode, we propose to follow Jiang’s approach in Ref. [2] to analyze the maximum strain to prevent fracture of the film structure.

The deformation maps shown in Fig. 3 are very important for the design of bio-integrated electronics, in the sense that they predict the buckling modes for any materials under any adhesion conditions. One crucial information they predict is the onset of interfacial delamination, indicated in these figures by the lower bounds of partial and total delamination modes (magenta, purple, cyan and red curves).

In this paper, an analytical model is established for thin-film on elastomer structures in the context of bio-integrated electronics applications. Under different conditions in interfacial adhesion, stiffness of the elastomer (tissues) and the levels of compressive strain, the thin film buckles into various modes. The transitions between these modes are predicted analytically, and summarized in four deformation maps. The lower bounds of the partial and total delamination modes predict the onset of interfacial delamination, which sets design criteria to avoid delamination and achieve intimate and conformal contact to bio-tissues. The analytically predicted information on deformation modes, maximum strain, and interfacial inactness, are important to the design and optimization of high performance bio-integrated electronics.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.taml.2015.11.010.

References


